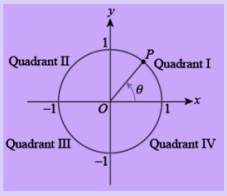
CHAPTER 6 : RATIOS AND GRAPHS OF TRIGONOMETRIC FUNCTIONS

BY : RRAEZZZ

6.1 THE VALUE OF SINE, COSINE AND TANGENT

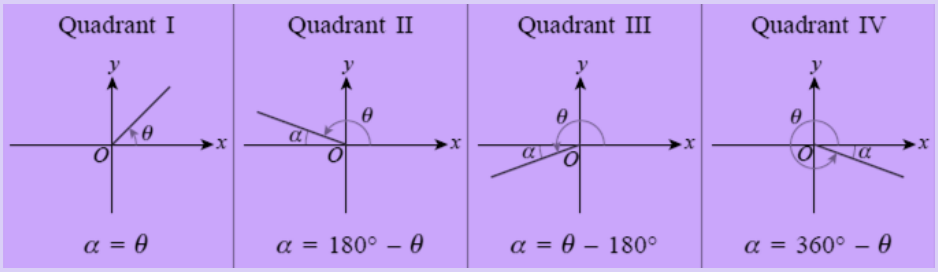
- ✓ A UNIT CIRCLE IS A CIRCLE THAT HAS A RADIUS OF 1 AND IS CENTRED ON THE ORIGIN.
- ✓ THE X-AXIS AND Y-AXIS DIVIDE INTO 4 EQUAL QUADRANTS, NAMELY QUADRANT I , QUADRANT II , QUADRANT III , AND QUADRANT IV

- IT IS GIVEN THAT P IS A POINT THAT MOVES ALONG THE CIRCUMFERENCE OF THE UNIT CIRCLE
- TETA IS THE ANGLE FORMED BY THE RADIUS OF THE UNIT CIRCLE, OP , FROM POSITIVE X-AXIS IN AN ANTICLOCKWISE DIRECTION



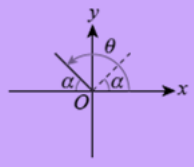
(a) point P is in quadrant I when $0° < \theta < 90°$,
(b) point P is in quadrant II when $90° < \theta < 180°$,
(c) point P is in quadrant III when $180° < \theta < 270°$,
(d) point P is in quadrant IV when $270° < \theta < 360°$.

- ✓ THE CORRESPONDING REFERENCE ANGLE , A , IS ALWAYS LESS THAN 90 DEGREE
- ✓ ANGLES IN QUADRANTS II, III AND IV HAVE CORRESPONDING REFERENCE ANGLES, A
- ✓ THE ANGLE IN QUADRANT I ITSELF IS THE CORRESPONDING REFERENCE ANGLE, A = 0



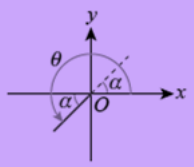
Quadrant II

$\sin\theta = +\sin\alpha = +\sin(180° - \theta)$
$\cos\theta = -\cos\alpha = -\cos(180° - \theta)$
$\tan\theta = -\tan\alpha = -\tan(180° - \theta)$



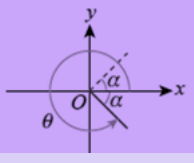
Quadrant III

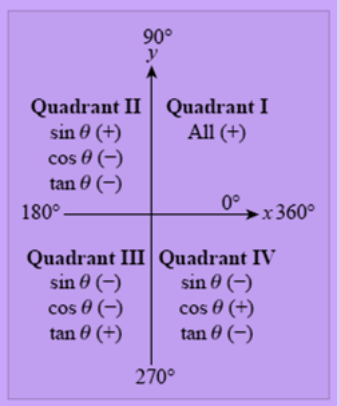
$\sin\theta = -\sin\alpha = -\sin(\theta - 180°)$
$\cos\theta = -\cos\alpha = -\cos(\theta - 180°)$
$\tan\theta = +\tan\alpha = +\tan(\theta - 180°)$



Quadrant IV

$\sin\theta = -\sin\alpha = -\sin(360° - \theta)$
$\cos\theta = +\cos\alpha = +\cos(360° - \theta)$
$\tan\theta = -\tan\alpha = -\tan(360° - \theta)$





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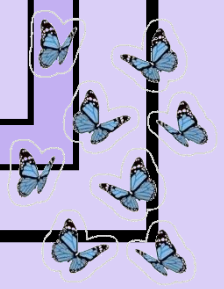
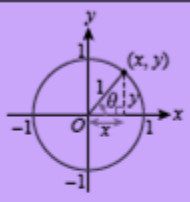
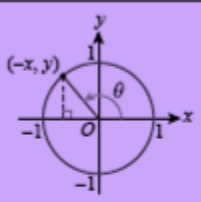
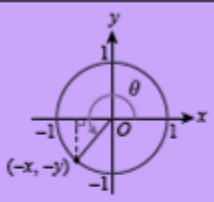


6.1 THE VALUE OF SINE, COSINE AND TANGENT

✓ IN GENERAL

The results of Mind Mobilisation 2 show that;

$\sin\theta$ = y-coordinate $\cos\theta$ = x-coordinate $\tan\theta = \frac{y\text{-coordinate}}{x\text{-coordinate}}$

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
			
$\sin\theta = +y$ $\cos\theta = +x$ $\tan\theta = +\frac{y}{x}$	$\sin\theta = +y$ $\cos\theta = -x$ $\tan\theta = -\frac{y}{x}$	$\sin\theta = -y$ $\cos\theta = -x$ $\tan\theta = +\frac{y}{x}$	$\sin\theta = -y$ $\cos\theta = +x$ $\tan\theta = -\frac{y}{x}$

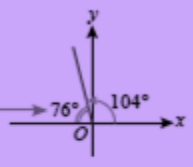
✓ DETERMINE VALUE OF SINE, COSINE AND TANGENT

(a) $\sin 104°$ (b) $\cos 208.2°$ (c) $\tan 318°17'$

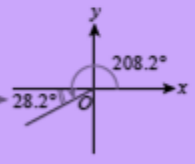
Solution:

(a) $\sin 104° = +\sin(180° - 104°)$ — sine values are positive in quadrant II
$= +\sin 76°$
$= 0.9703$

Corresponding reference angle



(b) $\cos 208.2° = -\cos(208.2° - 180°)$ — cosine values are negative in quadrant III
$= -\cos 28.2°$
$= -0.8813$



Corresponding reference angle

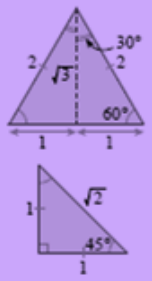
(c) $\tan 318°17' = -\tan(360° - 318°17')$ — tangent values are negative in quadrant IV
$= -\tan 41°43'$
$= -0.8915$

Smart TiPS

Quadrant I: All positive
Quadrant II: sin θ positive
Quadrant III: tan θ positive
Quadrant IV: cos θ positive

Tips to remember:
Use appropriate acronym.

MEMORY BOX



Ratio \ Angle	30°	60°	45°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

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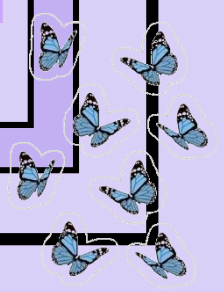
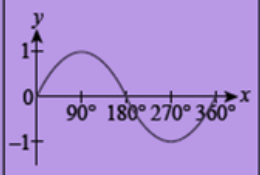
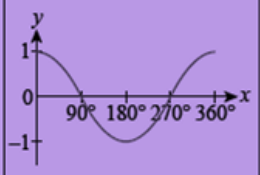
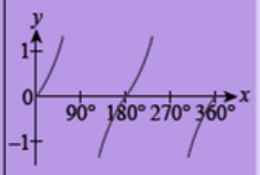
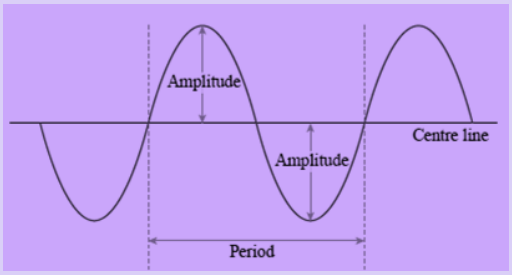


6.2 THE GRAPH OF SINE, COSINE AND TANGENT FUNCTION

✓ IN GENERAL

Shape of graph	$y = \sin x$	$y = \cos x$	$y = \tan x$
			
Maximum value	1	1	∞
Minimum value	−1	−1	−∞
x-intercept	0°, 180°, 360°	90°, 270°	0°, 180°, 360°
y-intercept	0	1	0

- PERIOD IS THE INTERVAL OF ONE COMPLETE CYCLE.
- THE TRIGONOMETRIC FUNCTION IS A PERIODIC FUNCTION.
- THE GRAPH OF A TRIGONOMETRIC FUNCTION REPEATS ITSELF ON A SPECIFIC INTERVAL.
- AMPLITUDE IS THE MAXIMUM DISTANCE MEASURED FROM THE CENTRE LINE.



✓ THE EFFECT OF CHANGES IN CONSTANTS A , B AND C ON THE GRAPH OF TIGONOMEGTRIC FUNCTION

	$y = a \sin bx + c$, $a > 0, b > 0$	$y = a \cos bx + c$, $a > 0, b > 0$	$y = a \tan bx + c$, $a > 0, b > 0$
The value of a changes	• Maximum and minimum values change • Amplitude of function = a		• Graph curvature changes • No amplitude
The value of b changes	• The period of the function changes • When the value of b increases, the graph appears to compress horizontally, the period of the function decreases • The period of sine and cosine functions = $\frac{360°}{b}$; The period of tangent function = $\frac{180°}{b}$		
The value of c changes	• The position of the graph changes • When $c > 0$, the graph moves c units vertically up the x-axis • When $c < 0$, the graph moves c units vertically down the x-axis		

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